Linear Functions as Models
Unit 2.5

Name: ___________________
Sec 4.4
Evaluating Linear Functions

<table>
<thead>
<tr>
<th>FORM A</th>
<th>FORM B</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = 5x - 3 )</td>
<td>( f(x) = 5x - 3 )</td>
</tr>
<tr>
<td>Find ( y ) when ( x = 2 )</td>
<td>Find ( f(2) ).</td>
</tr>
<tr>
<td>( y = 5 \times 2 - 3 )</td>
<td>( f(2) = 5 \times 2 - 3 )</td>
</tr>
<tr>
<td>( y = 10 - 3 )</td>
<td>( f(2) = 10 - 3 )</td>
</tr>
<tr>
<td>( y = 7 )</td>
<td>( f(2) = 7 )</td>
</tr>
<tr>
<td>solution: (( x, y )) = (2, 7)</td>
<td>solution: (( x, f(x) )) = (2, 7)</td>
</tr>
</tbody>
</table>

Compare and contrast the two examples above. You should notice you are doing the same steps in each form. You should also notice the two generate the same solution, or ordered pair. The difference between the two is the notation for the left side of the equations. Form A uses \( y \), while Form B uses \( f(x) \). This \( f(x) \) notation is called function notation and can be used with functions as a substitute for \( y \).

1. Suppose that \( f(x) = 7x - 3 \)
   a. Find \( f(3) \)
   b. Find \( f(-5) \)

2. Suppose \( h(x) = -2x + 10 \)
   a. Find \( h(-8) \)
   b. Find \( h(5) \)

3. Suppose that \( f(x) = \frac{3}{4}x + 1 \)
   a. Find \( f(16) \)
   b. Find \( f(-5) \)

4. Suppose \( h(x) = -\frac{5}{2}x + 4 \)
   a. Find \( h(4) \)
   b. Find \( h(0) \)
The first side of this sheet has you finding values for $y$ or $f(x)$, when given an $x$ value. This notation can also be used when given the value of $f(x)$ (think: given a value for $y$, find the value for $x$).

<table>
<thead>
<tr>
<th>FORM A</th>
<th>FORM B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = 5x - 3$</td>
<td>$f(x) = 5x - 3$</td>
</tr>
<tr>
<td>Find $x$ when $y = 2$</td>
<td>Find $x$ when $f(x) = 2$</td>
</tr>
<tr>
<td>$y = 5x - 3$</td>
<td>$f(x) = 5x - 3$</td>
</tr>
<tr>
<td>$2 = 5x - 3$</td>
<td>$2 = 5x - 3$</td>
</tr>
<tr>
<td>$5 = 5x$</td>
<td>$5 = 5x$</td>
</tr>
<tr>
<td>$1 = x$</td>
<td>$1 = x$</td>
</tr>
<tr>
<td>solution: $(x, y) = (1, 2)$</td>
<td>solution: $(x, f(x)) = (1, 2)$</td>
</tr>
</tbody>
</table>

1. Suppose that $f(x) = 7x - 3$
   Find $x$ when $f(x) = 3$

2. Suppose $h(x) = -2x + 10$
   Find $x$ when $h(x) = -8$

3. Suppose that $f(x) = \frac{3}{4} x + 1$
   Find $x$ when $f(x) = 19$

4. Suppose $h(x) = -3x + 10$
   Find $x$ when $h(x) = -2$
Sec 3.5 Mathematical Models

**Goal:** Given a situation in which two real-world Variables are related by a straight-line graph, Be able to:
  a) sketch a graph  
  b) write the equation  
  c) use equation to predict values  
  d) determine slope and intercepts with their meaning in the real world

**WARM-UP**

For each of the following, write the independent variable and dependent variable:

- A local donut establishment charges 20 cents each for a donut. The amount charged is a function of donuts bought.

- A computer store sells 10 floppy diskettes for $15, and 30 diskettes for $40. Assume that the number of dollars varies linearly with the number of diskettes.

- Phoebe Small still has 35 pages of history to be read after she has been reading for 10 minutes, and 5 pages left after she has been reading for 50 minutes. Assume that the number of pages left to read varies linearly with the number of minutes she has been reading.
1. **Bathtub Problem**: You pull out the plug from the bathtub. After 40 seconds, there are 13 gallons of water left in the tub. One minute after you pull the plug, there are 10 gallons left. Assume that the number of gallons varies linearly with the time since the plug was pulled.

a. Write the particular equation expressing the number of gallons (g) left in the tub in terms of the number of seconds (s) since you pulled the plug.

b. How many gallons would be left after 20 seconds? 50 seconds?

c. At what time will there be 7 gallons left in the tub?

d. Find the y-intercept (gallon-intercept). What does this number represent in the real world?

e. Find the x-intercept (time-intercept). What does this number represent in the real world?

f. Plot the graph of this linear function. Use a suitable domain.

```
Time in seconds

Water in gallons

10 gallons
```

g. What is the slope? What does this number represent?
2. **Driving Home Problem:** As you drive home from the football game, the number of kilometers you are away from home depends on the number of minutes you have been driving. Assume that the distance varies linearly with time. Suppose you are 11 km from home when you have been driving for 10 minutes, and 8 km from home when you have been driving for 15 minutes.

a. Write the particular equation expressing the number of kilometers you are from home (d) in terms of the number of minutes since you left the game (t).

b. Predict your distance from home after driving for 20 min., 25 min., and 30 min.

c. When were you 7 km from home, how many minutes have you been traveling?

d. Find the distance-intercept. What does this number represent in the real world?

e. Find the time-intercept. What does this number represent in the real world?

f. Plot the graph of this linear function. Use a suitable domain.

![Graph of linear function](image)

g. What is the slope? What does this number represent? What is the significance that the slope is negative?
3. **Cricket Problem:** Based on information in *Deep River Jim’s Wilderness Trailbook*, the rate at which crickets chirp is a linear function of temperature. At 59°F they make 76 chirps per minute, and at 65°F they make 100 chirps per minute.

a. Write the particular equation expressing chirping rate (c) in terms of temperature (t).

b. Predict the chirping rate for 90°F.

c. How warm is it if you count 120 chirps per minute?

d. Find the chirping-rate intercept. What does it tell you about the real world?

e. Calculate the temperature-intercept. What does this number represent in the real world?

f. Plot the graph of this linear function. Use a suitable domain and label the horizontal and vertical axis.

  
g. What is the slope? What does this number represent?
4. **Cost of Owning a Car Problem:** The number of dollars per month it costs you to own a car is a function of the number of kilometers per month you drive it. Based on information in an issue of *Time* magazine, the cost varies linearly with the distance, and is $366 per month for 300 km per month, and $510 per month for 1500 km per month.

a. Write the particular equation expressing cost (c) in terms of distance (d).

b. Predict your monthly cost if you drive 1000 km/month, then for 2000 km/month.

c. How far could you drive without exceeding a monthly cost of $600?

d. Find the cost-rate intercept. What does it tell you about the real world?

e. Calculate the distance-intercept. What does this number represent in the real world?

f. Plot the graph of this linear function. Use a suitable domain and label the horizontal and vertical axis.

g. What is the slope? What does this number represent?
5. **Shoe Size Problem:** The size of a shoe a person needs varies linearly with the length of his or her foot. The smallest adult shoe size is Size 5, and fits a 9-inch long foot. An 11-inch long foot takes a Size 11 shoe.

a. Write the particular equation expressing shoe size \( s \) in terms of foot length \( l \).

b. If your foot is a foot long, what size do you need?

c. Bob Lanier, who once played in the NBA wears a Size 22. How long is his foot?

d. Find the shoe-size-rate intercept. What does it tell you about the real world?

e. Calculate the foot-length-intercept. What does this number represent in the real world?

f. Plot the graph of this linear function. Use a suitable domain and label the horizontal and vertical axis.

g. What is the slope? What does this number represent?
6. **Speeding Bullet Problem:** The speed a bullet is traveling depends on the number of feet the bullet has traveled since it left the gun. The bullet is traveling at 3500 ft./sec. when it is 25 feet from the gun, and at 2600 ft./sec., it is 250 feet away.

a. Write the particular equation expressing the speed of the bullet (s) in terms of its distance from the gun (d).

b. How fast is the bullet going when it has traveled 300 ft.?  
c. How far has the bullet traveled when it has slowed to 500 ft./sec?

d. What is the speed of the bullet immediately after the trigger is pulled?  
e. When the bullet has stopped, how far has it traveled?

f. Plot the graph of this linear function. Use a suitable domain and label the horizontal and vertical axis.

g. What is the slope? What does this number represent?
7. **Taxi Problem:** To take a taxi in downtown St. Louis, it will cost you $3.00 to go a mile. After 6 miles, it will cost $5.25. The cost varies linearly with the distance traveled.

a. Write the particular equation expressing cost (c) in terms of miles (d) traveled.

b. How much will it cost you to travel 10 miles in a taxi? c. How many miles can you travel if you only have $20 to spend?

d. Calculate the cost-intercept. What does this number represent in the real world?

e. Plot the graph of this linear function. Use a suitable domain and label the horizontal and vertical axis.

f. What is the slope? What does this number represent?
8. **Milk Problem:** The Magic Market sells one-gallon cartons of milk (4 quarts) for $3.09 each and half-gallon (2 quarts) cartons for $1.65 each. Assume that the number of cents you pay for a carton of milk varies linearly with the number of quarts the carton holds.

a. Write the particular equation expressing price \( p \) in terms of quarts \( q \).

b. If the Magic Market sold 3-gal. cartons, what would your equation predict the price to be?

c. Suppose you found cartons of milk marked at $3.45, but there is nothing on the carton to tell what size it is. According to your equation, how much would such a carton hold?

d. Plot the graph of this linear function. Use a suitable domain and label the horizontal and vertical axis.

e. What is the slope? What does this number represent?

f. The actual prices for pint cartons (1/2 quart) and one-quart cartons are $0.57 and $0.99 respectively. Do these prices fit your mathematical model? If not, are they higher than predicted, or lower?
9. **Celsius-to-Fahrenheit Problem:** The Fahrenheit temperature, “F”, and the Celsius temperature “C,” of an object are related by a linear function. Water boils at 100˚C or 212˚F, and water freezes at 0˚C or 32˚F.

a. Write an equation expressing $F$ in terms of $C$.

b. Lead boils at 1620˚C. What Fahrenheit temperature is this?

c. Normal human body temperature is 98.6˚F. What Celsius temperature is this?

d. If the weather forecaster says it will be 40˚C today, will it be hot, cold or medium. Explain.

e. The coldest possible temperature is absolute zero, -273˚C. What Fahrenheit temperature is this?

f. Plot the graph of this linear function. Use a suitable domain and label the horizontal and vertical axis.

g. Transform the equation so that $C$ is in terms of $F$. 
10. **Thermal Expansion Problem:** Bridges on highways often have expansion joints, which are small gaps in the roadway between one bridge section and the next. The gaps are put there so the bridge will have room to expand when the weather gets hot. Suppose a bridge has a gap of 1.3 cm when the temperature is 22°C, and that the gap narrows to 0.9 cm when the temperature warms to 30°C. Assume the gap width varies linearly with the temperature.

   a. Write the particular equation for gap width (w) in terms of temperature (t).

   b. How wide would the gap be at 35°C?
      At -10°C?

   c. At what temperature would the gap close completely? What mathematical name is given to this temperature?

   d. What is the width-intercept? What does this tell you in the real world?

   e. Plot the graph of this linear function. Use a suitable domain and label the horizontal and vertical axis.

   f. What is the slope? What does this number represent?